Sound Field Recording Using Distributed Microphones Based On Harmonic Analysis of Infinite Order

無限次元調和解析に基づく 分散配置マイクロフォンを用いた音場計測

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  - 主な内容：
    - N. Ueno, S. Koyama, and H. Saruwatari, "Sound field recording using distributed microphones based on harmonic analysis of infinite order," *IEEE Signal Process. Letters*, vol. 25, no. 1, pp. 135-139, 2018.

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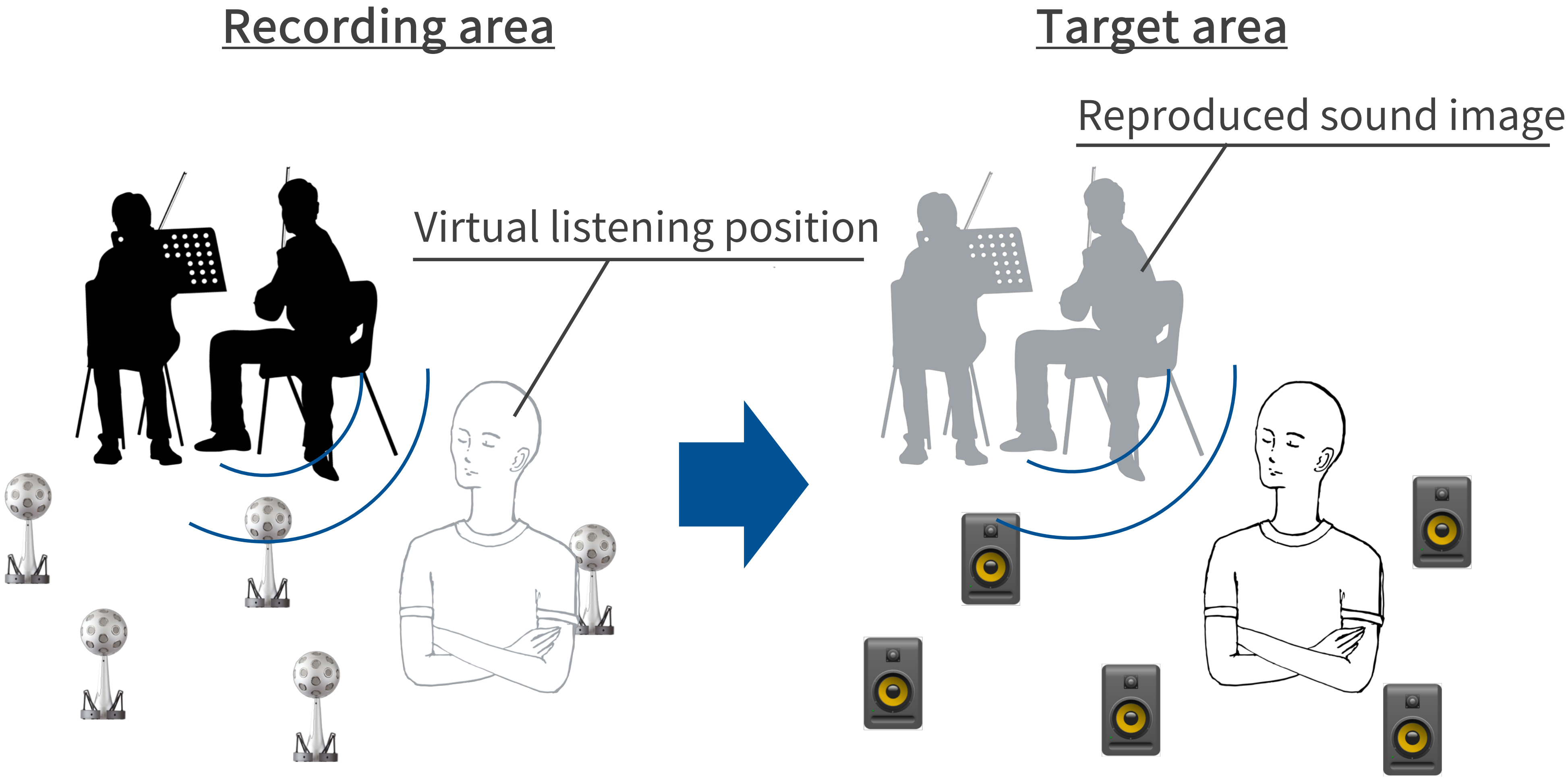
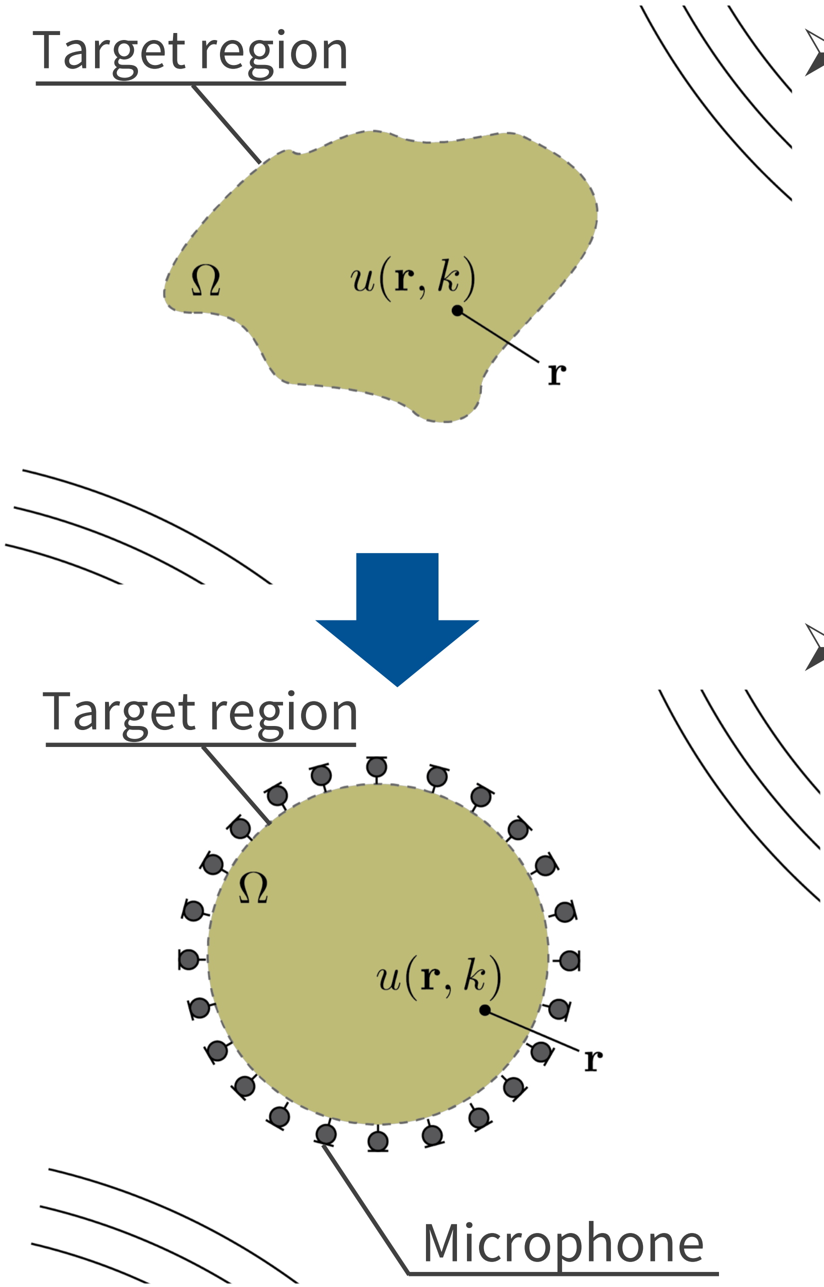
Spatial Sound Recording and Reproduction

How to capture and reproduce physically correct sound field?



Conventional: Boundary Integral Eq.

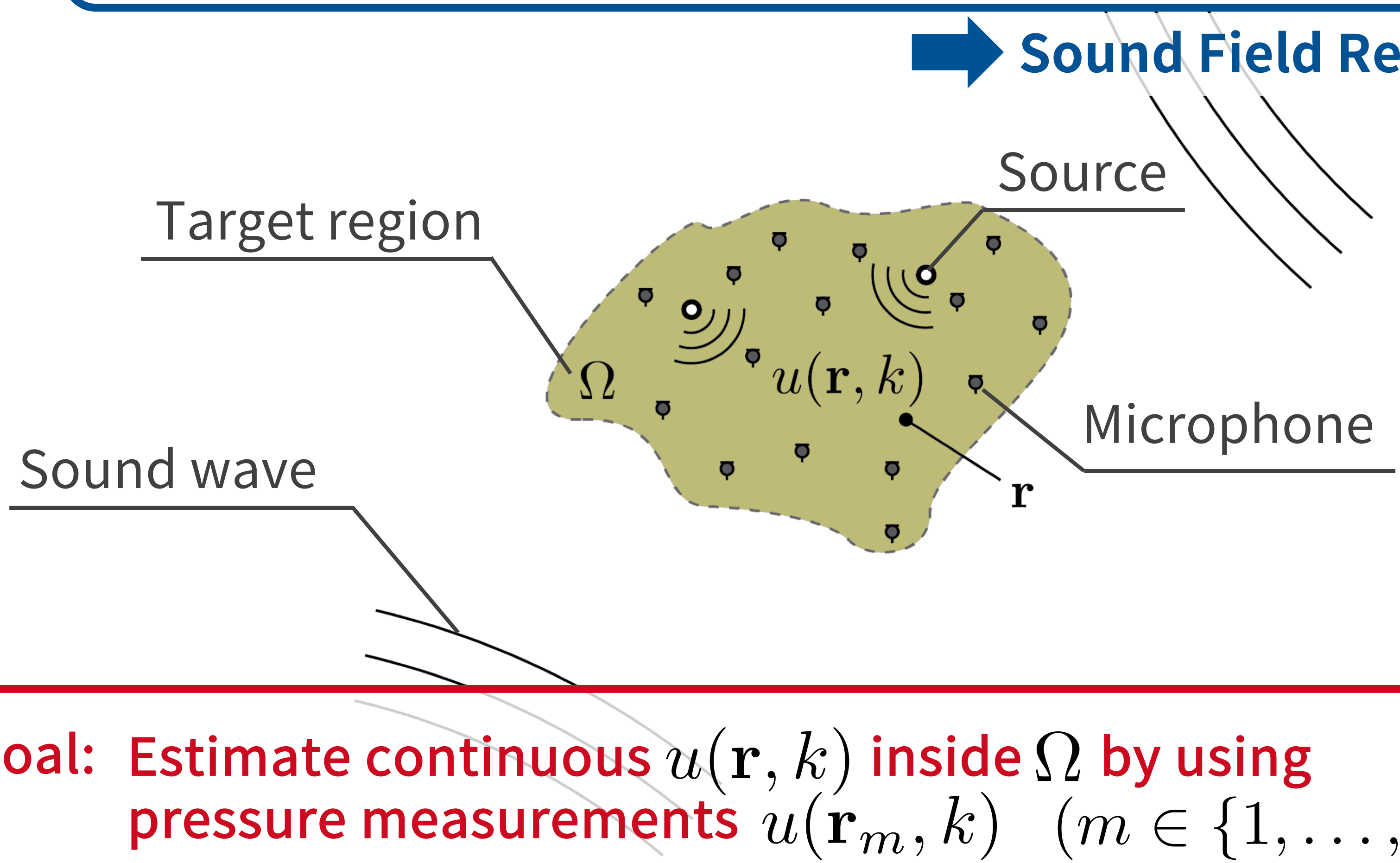


- Boundary-integral-based representation (Kirchhoff-Helmholtz eq.)
  - Pressure and its gradient on boundary are required to estimate inside target region
- Simplified shape of target region to estimate only by pressure distribution (e.g. spherical shape)
  - Pressure mics on boundary surface
  - Rigid baffle / directional mics to avoid forbidden freq problem
  - **No flexibility in array geometry 😱**
  - **Large target region requires large array size 😱**

Today's Topic

How to estimate and interpolate continuous sound field from measurements of distributed microphones?

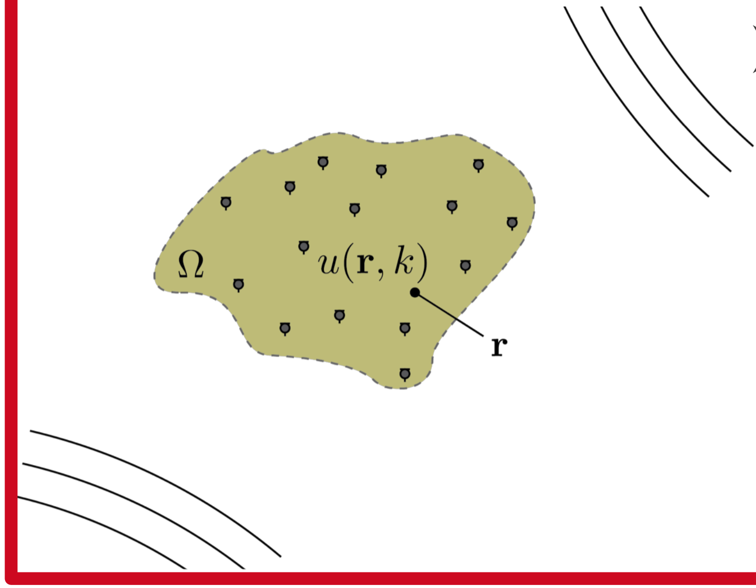
➡ **Sound Field Reconstruction**



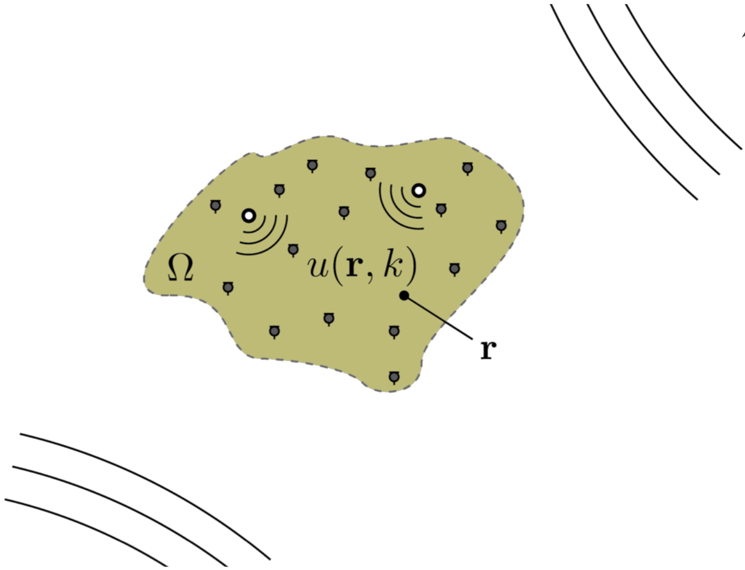
Goal: Estimate continuous $u(\mathbf{r}, k)$ inside Ω by using pressure measurements $u(\mathbf{r}_m, k) \quad (m \in \{1, \ldots, M\})$

➡ Visualization, reproduction by loudspeakers/headphones etc…

Sound Field Reconstruction

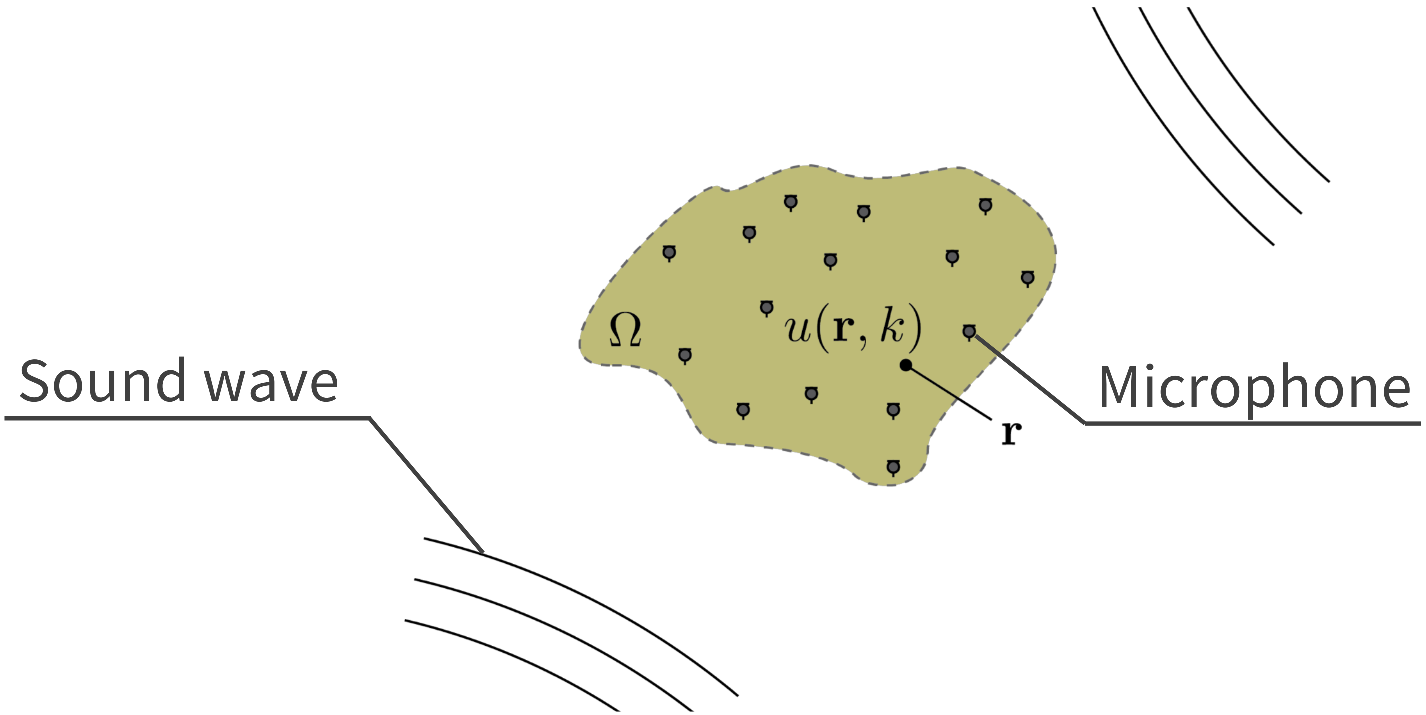


- **Target region does NOT include any sources**
  - Interpolation with constraint of homogeneous Helmholtz eq.
  - Decomposition of captured sound field into plane-wave or harmonic functions
  - [Ueno+ IEEE SPL 2018, IWAENC 2018]



- **Target region includes some sources**
  - <u>ill-posed problem</u>!
  - Some assumptions must be imposed on source distribution
  - [Koyama+ JASA 2018, IEEE JSTSP 2019], [Murata+ IEEE TSP 2018], [Takida+ IEEE SAM 2018, ICASSP 2019]

Homogeneous Sound Field Reconstruction



- Sound field inside source-free region

➡ $u(\mathbf{r}, k)$ **satisfies homogeneous Helmholtz eq.**

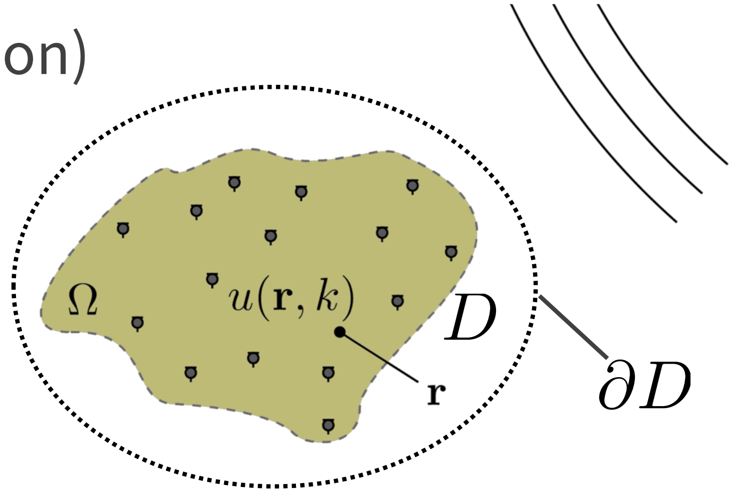
$$\left\{ \begin{array}{l} (\nabla^2 + k^2) u(\mathbf{r}, k) = 0 \\ \text{Unknown boundary condition on room surface} \end{array} \right.$$

Sound Field Decomposition

Decomposition into element solutions of Helmholtz eq.
➡ Reconstruction with constraint of Helmholtz eq.

- Plane-wave function (Herglotz wave function)

$$u(\mathbf{r}) = \int_{\boldsymbol{\eta} \in \mathbb{S}^2} \gamma(\boldsymbol{\eta}) \underline{e^{jk\langle \mathbf{r}, \boldsymbol{\eta} \rangle}} \mathrm{d}\boldsymbol{\eta}$$



- Spherical wave function

$$u(\mathbf{r}) = \sum_{\nu=0}^{\infty} \sum_{\nu=-\mu}^{\mu} \alpha_{\nu}^{\mu} \underline{j_{\nu}(kr) Y_{\nu}^{\mu}(\theta, \phi)}$$

- Equivalent source method (single layer potential)

$$u(\mathbf{r}) = \int_{\mathbf{r}' \in \partial D} \psi(\mathbf{r}') \underline{G(\mathbf{r}|\mathbf{r}')} \mathrm{d}\mathbf{r}'$$

Free-field Green's func.: $G(\mathbf{r}|\mathbf{r}') = \dfrac{e^{jk\|\mathbf{r}-\mathbf{r}'\|_2}}{4\pi\|\mathbf{r}-\mathbf{r}'\|_2}$

[Koopmann+ 1989, Colton+ 2013]

Idea

- Spherical wave function expansion w/ truncation

[Laborie+ 2003, Samarasinghe+ 2014]

$$u(\mathbf{r}) \approx \sum_{\nu=0}^{N} \sum_{\nu=-\mu}^{\mu} \alpha_{\nu}^{\mu}(\mathbf{r}_0) j_{\nu}(kr^{(0)}) Y_{\nu}^{\mu}(\theta^{(0)}, \phi^{(0)})$$

Expansion center

  - Estimate expansion coefs by solving linear eq of coefs $\alpha_{\nu}^{\mu}(\mathbf{r}_0)$ and measurement values $u(\mathbf{r}_m)$
  - Empirical truncation of expansion order 😭
  - Setting of expansion center $\mathbf{r}_0$ is necessary 😭

- Spherical wave function expansion w/ infinite order

[Ueno+ IEEE SPL 2018]

$$u(\mathbf{r}) = \sum_{\nu=0}^{\infty} \sum_{\nu=-\mu}^{\mu} \alpha_{\nu}^{\mu}(\mathbf{r}_0) j_{\nu}(kr^{(0)}) Y_{\nu}^{\mu}(\theta^{(0)}, \phi^{(0)})$$

  - Leading to simple solution 😎
  - No more empirical setting for truncation and expansion center 😎
  - (Spatial error estimation by introducing Bayesian formulation 😎)

Harmonic Analysis of Infinite Orders

[Ueno+ IEEE SPL 2018]

- Spherical wave function expansion

$$u(\mathbf{r}) = \sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} \alpha_\nu^\mu(\mathbf{r}_0)\varphi_\nu^\mu(\mathbf{r}-\mathbf{r}_0)$$

$$\varphi_\nu^\mu(\mathbf{r}-\mathbf{r}_0) = \sqrt{4\pi}\underline{j_\nu(kr^{(0)})Y_\nu^\mu(\theta^{(0)},\phi^{(0)})}$$

Spherical Bessel func x Spherical harmonic func

(Here, $\mathbf{r}-\mathbf{r}_0 = (r^{(0)}, \theta^{(0)}, \phi^{(0)})$)

- Representation by infinite vectors

$$u(\mathbf{r}) = \boldsymbol{\alpha}(\boldsymbol{r}_0)^{\mathsf{T}}\boldsymbol{\varphi}(\boldsymbol{r}-\boldsymbol{r}_0)$$

– Coefficient vector: $\boldsymbol{\alpha}(\mathbf{r}_0) \in \mathbb{C}^\infty$
– Basis-function vector: $\boldsymbol{\varphi}(\mathbf{r}-\mathbf{r}_0) \in \mathbb{C}^\infty$

Harmonic Analysis of Infinite Orders

[Ueno+ IEEE SPL 2018]

- Measurement by m th mic at $\mathbf{r}_m$ with directivity of $c_m(\theta, \phi)$ is represented as

Measurement noise

$$s_m = \sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} c_{m,\nu}^{\mu *} \alpha_\nu^\mu(\mathbf{r}_0) + \epsilon_m$$

Expansion coef of $c_m(\theta, \phi)$

$$= \mathbf{c}_m^{\mathsf{H}} \boldsymbol{\alpha}(\mathbf{r}_m) + \epsilon_m$$: Representation by infinite vectors

$$= \mathbf{c}_m^{\mathsf{H}} \mathbf{T}(\mathbf{r}_m - \mathbf{r}_0) \boldsymbol{\alpha}(\mathbf{r}_0) + \epsilon_m$$

Translation matrix from $\mathbf{r}_0$ to $\mathbf{r}_m$ [Martin 2006]

- Stacking M measurements: $\mathbf{s} = [s_1, \ldots, s_M]^{\mathsf{T}}, \ \boldsymbol{\epsilon} = [\epsilon_1, \ldots, \epsilon_M]^{\mathsf{T}}$

$$\mathbf{s} = \boldsymbol{\Xi}(\mathbf{r}_0)^{\mathsf{H}} \boldsymbol{\alpha}(\mathbf{r}_0) + \boldsymbol{\epsilon}$$

$$\boldsymbol{\Xi}(\mathbf{r}_0) = [(\mathbf{c}_1^{\mathsf{H}} \mathbf{T}(\mathbf{r}_1 - \mathbf{r}_0))^{\mathsf{H}}, \ldots, (\mathbf{c}_M^{\mathsf{H}} \mathbf{T}(\mathbf{r}_M - \mathbf{r}_0))^{\mathsf{H}}]$$
$$= [\mathbf{T}(\mathbf{r}_1 - \mathbf{r}_0)\mathbf{c}_1, \ldots, \mathbf{T}(\mathbf{r}_M - \mathbf{r}_0)\mathbf{c}_M]$$

Harmonic Analysis of Infinite Orders

[Ueno+ IEEE SPL 2018]

- Expansion coef at $\mathbf{r}_0$ is estimated as

$$\hat{\alpha}(\mathbf{r}_0) = \boldsymbol{\Xi}(\mathbf{r}_0)(\boldsymbol{\Psi} + \lambda\mathbf{I})^{-1}\mathbf{s}$$

$$\begin{aligned}(\boldsymbol{\Psi})_{m,m'} &= \left(\boldsymbol{\Xi}(\mathbf{r}_0)^{\mathsf{H}}\boldsymbol{\Xi}(\mathbf{r}_0)\right)_{m,m'} \\ &= \mathbf{c}_m^{\mathsf{H}}\mathbf{T}(\mathbf{r}_m - \mathbf{r}_0)\mathbf{T}(\mathbf{r}_0 - \mathbf{r}_{m'})\mathbf{c}_{m'} \\ &= \mathbf{c}_m^{\mathsf{H}}\mathbf{T}(\mathbf{r}_m - \mathbf{r}_{m'})\mathbf{c}_{m'}\end{aligned}$$

➡ Dependency on expansion center $\mathbf{r}_0$ is removed

- Expansion coef at arbitrary position $\mathbf{r}$:

$$\hat{\alpha}(\mathbf{r}) = \boldsymbol{\Xi}(\mathbf{r})(\boldsymbol{\Psi} + \lambda\mathbf{I})^{-1}\mathbf{s}$$

Expansion coef at arbitrary position can be estimated independently of truncation and expansion center

Conventional Harmonic Analysis

[Laborie+ 2003, Samarasinghe+ 2014]

- Spherical wave function expansion w/ truncation

$$u(\mathbf{r}) \approx \sum_{\nu=0}^{N} \sum_{\mu=-\nu}^{\nu} \alpha_\nu^\mu(\mathbf{r}_0)\varphi_\nu^\mu(\mathbf{r}-\mathbf{r}_0)$$: approx. by truncation

- Linear eq of expansion coef and measurements

$$\mathbf{s} = \bar{\boldsymbol{\Xi}}(\mathbf{r}_0)^{\mathsf{H}}\bar{\boldsymbol{\alpha}}(\mathbf{r}_0) + \boldsymbol{\epsilon}$$

$$\begin{cases} \bar{\boldsymbol{\alpha}}(\mathbf{r}_0) \in \mathbb{C}^{(N+1)^2} \\ \bar{\boldsymbol{\Xi}}(\mathbf{r}_0) \in \mathbb{C}^{(N+1)^2 \times M} \end{cases}$$

- Estimate of expansion coef at $\mathbf{r}$

$$\hat{\boldsymbol{\alpha}}(\mathbf{r}) = \bar{\mathbf{T}}(\mathbf{r}-\mathbf{r}_0)\underbrace{\bar{\boldsymbol{\Xi}}(\mathbf{r}_0)\left(\bar{\boldsymbol{\Xi}}(\mathbf{r}_0)^{\mathsf{H}}\bar{\boldsymbol{\Xi}}(\mathbf{r}_0) + \lambda\mathbf{I}\right)^{-1}\mathbf{s}}_{\text{Estimate of } \bar{\boldsymbol{\alpha}}(\mathbf{r}_0)}$$

Setting of appropriate truncation order and expansion center is necessary

Relation to kernel ridge regression

[Ueno+ IEEE SPL 2018, IWAENC 2018]

- Harmonic analysis of infinite orders for pressure microphone case ($c_{m,\nu}^{\mu} = \delta_{\nu,0}\delta_{\mu,0}$)

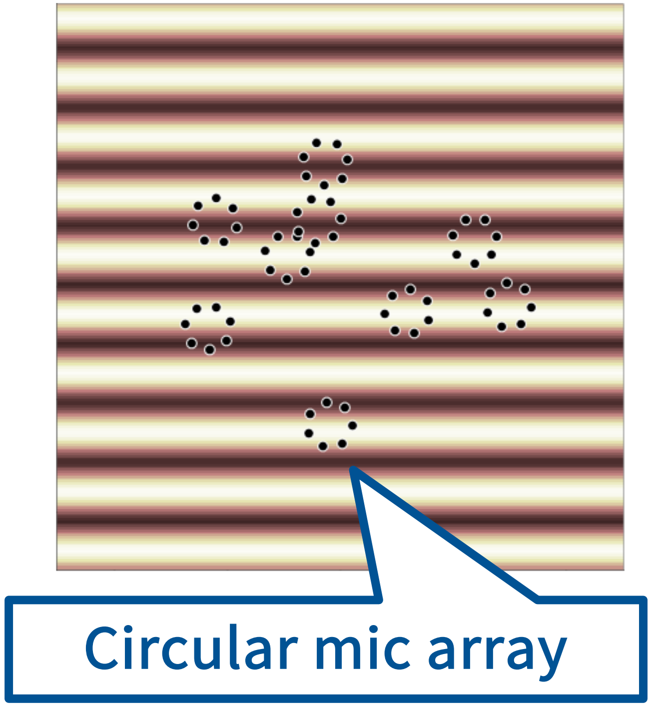
$$
\begin{aligned}
\hat{u}(\mathbf{r}) &= \boldsymbol{\varphi}(\mathbf{r}-\mathbf{r}_0)^{\mathsf{T}}\boldsymbol{\Xi}(\mathbf{r}_0)(\boldsymbol{\Psi}+\lambda\mathbf{I})^{-1}\mathbf{s} \\
&= \sum_{m=1}^{M} \left((\boldsymbol{\Psi}+\lambda\mathbf{I})^{-1}\mathbf{s}\right)_m j_0(k\|\mathbf{r}-\mathbf{r}_m\|_2)
\end{aligned}
$$

$$
\boldsymbol{\Psi} = \begin{bmatrix} j_0(k\|\mathbf{r}_1-\mathbf{r}_1\|) & \cdots & j_0(k\|\mathbf{r}_1-\mathbf{r}_M\|) \\ \vdots & \ddots & \vdots \\ j_0(k\|\mathbf{r}_M-\mathbf{r}_1\|) & \cdots & j_0(k\|\mathbf{r}_M-\mathbf{r}_M\|) \end{bmatrix}
$$

Correspond to kernel ridge regression with kernel function of 0th-order spherical Bessel function

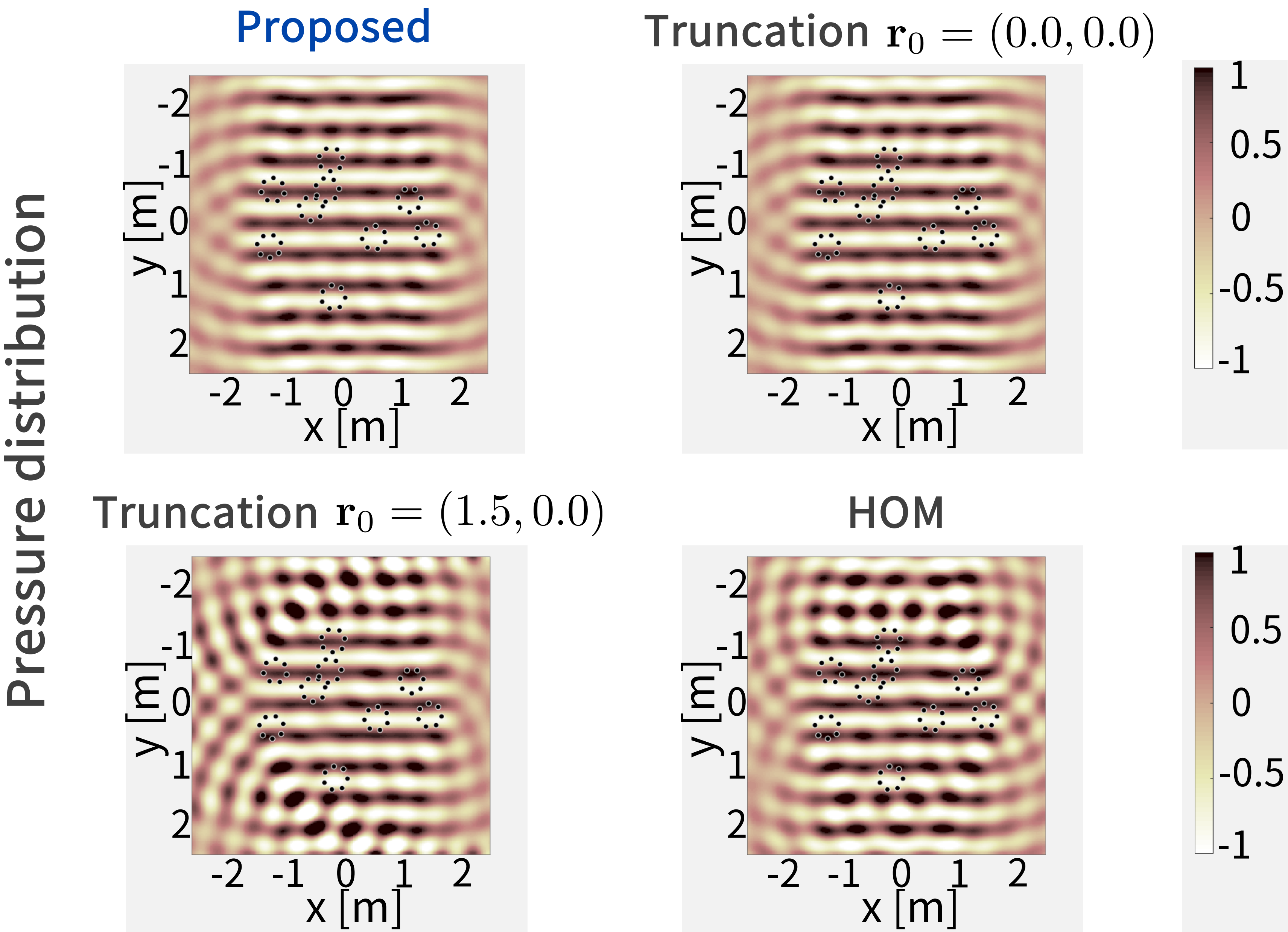
Experiments

- Reconstruction of plane waves in 2D sound field using distributed circular arrays
  - 9 circular arrays are randomly placed inside circular region (1.5 m radius)
  - 7 equiangularly-spaced mics for each circular array (0.2 m radius)
  - Monotonic plane wave
  - Gaussian noise of SNR 20 dB

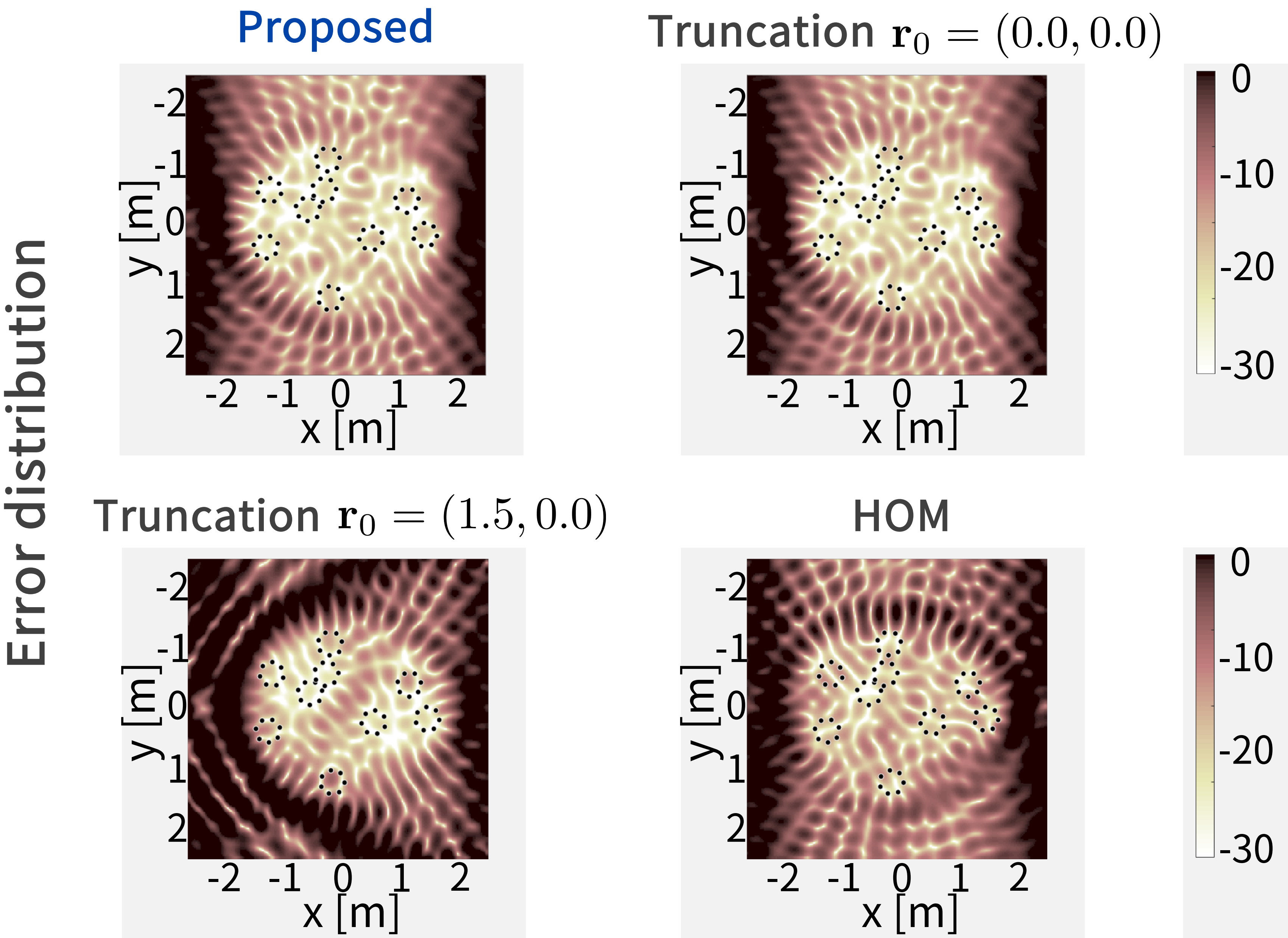


- Performance comparison
  - **Proposed**
  - **Truncation** [Laborie+ 2003]
  - **HOM** [Samarasinghe+ 2014]

Results at 650 Hz

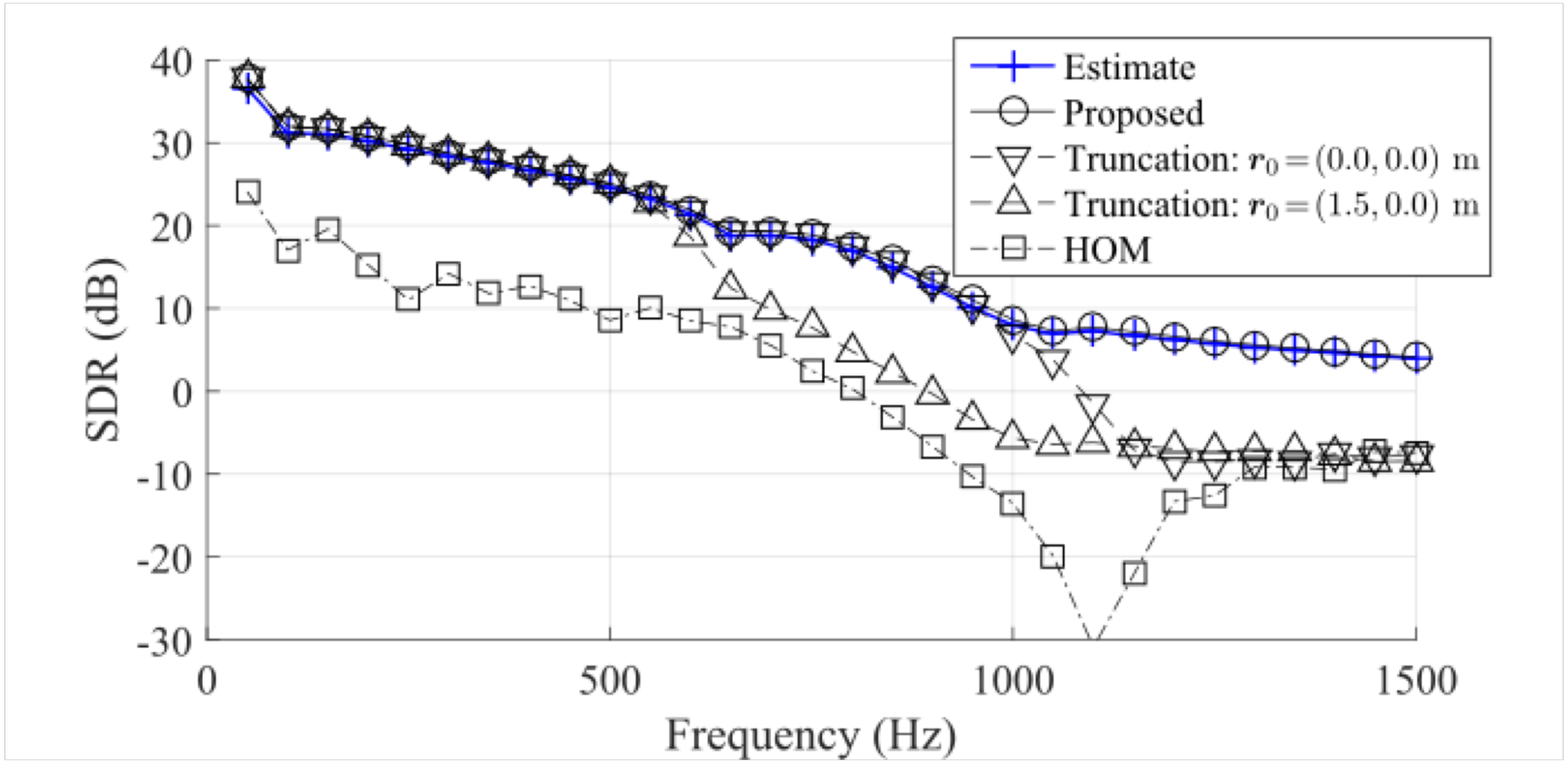


Results at 650 Hz



Frequency vs. SDR

- Average SDR for 16 array geometries and 64 plane-wave angles



Signal-to-distortion ratio (SDR):

True pressure

$$\mathrm{SDR}(\omega) = 10 \log_{10} \frac{\int_\Omega |u(\boldsymbol{r},\omega)|^2 \,\mathrm{d}\boldsymbol{r}}{\int_\Omega |u(\boldsymbol{r},\omega) - \hat{u}(\boldsymbol{r},\omega)|^2 \,\mathrm{d}\boldsymbol{r}} \text{ (dB)}$$

Estimated pressure

Conclusion

- Sound field reconstruction using distributed mics
  - Homogeneous and inhomogeneous sound fields
  - Decomposition into element solutions of Helmholtz eq.
  - Spherical wave function expansion of infinite orders
  - Expansion coef estimation at arbitrary position independently of truncation order and expansion center
  - Spatial error estimation by using posterior covariance
  - Numerical experiments in 2D
    - High reconstruction accuracy at high freqs
    - Good fit to spatial error estimation

Thank you for your attention!

Related publications (1/2)

- S. Koyama and L. Daudet, "Sparse representation of a spatial sound field in a reverberant environment," *IEEE J.STSP*, 2019. (in press)
- H. Ito, S. Koyama, *et al.* "Feedforward spatial active noise control based on kernel interpolation of sound field," *Proc. IEEE ICASSP*, Brighton, May 2019. (to appear)
- Y. Takida, S. Koyama, N. Ueno, and H. Saruwatari, "Robust gridless sound field decomposotion based on structured reciprocity gap functional in spherical harmonic domain," *Proc. IEEE ICASSP*, Brighton, May 2019. (to appear)
- N. Ueno, S. Koyama, and H. Saruwatari, "Kernel ridge regression with constraint of Helmholtz equation for sound field interpolation," *Proc. IWAENC*, 2018.
- Y. Takida, S. Koyama, and H. Saruwatari. "Exterior and interior sound field separation using convex optimization: comparison of signal models," *Proc. EUSIPCO*, 2018.
- Y. Takida, S. Koyama, *et al.* "Gridless sound field decomposition based on reciprocity gap functional in spherical harmonic domain," *Proc. IEEE SAM*, 2018.
- N. Ueno, S. Koyama, and H. Saruwatari, "Sound field reproduction with exterior radiation cancellation using analytical weighting of harmonic coefficients," *Proc. IEEE ICASSP*, 2018.
- S. Koyama, *et al.* "Sparse sound field decomposition for super-resolution in recording and reproduction," *JASA*, 2018.
- N. Murata, S. Koyama, *et al.* "Sparse representation using multidimensional mixed-norm penalty with application to sound field decomposition," *IEEE Trans. Signal Process.*, 2018.
- N. Ueno, S. Koyama, and H. Saruwatari, "Sound field recording using distributed microphones based on harmonic analysis of infinite order," *IEEE Signal Process. Letters*, 2018.
- S. Koyama and L. Daudet, "Comparison of reverberation models for sparse sound field decomposition," *Proc. IEEE WASPAA*, 2017.
- S. Koyama, *et al.* "Effect of multipole dictionary in sparse sound field decomposition for super-resolution in recording and reproduction," *Proc. ICSV*, 2017.
- N. Ueno, S. Koyama, and H. Saruwatari, "Listening-area-informed sound field reproduction with Gaussian prior based on circular harmonic expansion," *Proc. HSCMA*, 2017.

Related publications (2/2)

- N. Murata, S. Koyama, *et al.* "Spatio-temopral sparse sound field decomposition considering acoustic source signal characteristics," *Proc. IEEE ICASSP*, 2017.
- N. Ueno, S. Koyama, and H. Saruwatari, "Listening-area-informed sound field reproduction based on circular harmonic expansion," *Proc. IEEE ICASSP*, 2017.
- S. Koyama and H. Saruwatari, "Sound field decomposition in reverberant environment using sparse and low-rank signal models," *Proc. IEEE ICASSP*, 2016.
- N. Murata, S. Koyama, *et al.* "Sparse sound field decomposition with multichannel extension of complex NMF," *Proc. IEEE ICASSP*, 2016.
- S. Koyama, *et al.* "Structured sparse signal models and decomposition algorithm for super-resolution in sound field recording and reproduction," *Proc. IEEE ICASSP*, 2015.
- S. Koyama, *et al.* "Sparse sound field representation in recording and reproduction for reducing spatial aliasing artifacts," *Proc. IEEE ICASSP*, 2014.
- S. Koyama, *et al.* "Analytical approach to transforming filter design for sound field recording and reproduction using circular arrays with a spherical baffle," *JASA*, 2016.
- S. Koyama, *et al.* "Source-location-informed sound field recording and reproduction," *IEEE J. Sel. Topics Signal Process.*, 2015.
- J. Trevino, S. Koyama, *et al.* "Mixed-order ambisonics encoding of cylindrical microphone array signals," *Acoust. Sci. Tech.*, *Acoust. Letter*, 2014.
- S. Koyama, *et al.* "Wave field reconstruction filtering in cylindrical harmonic domain for with-height recording and reproduction," *IEEE/ACM Trans. ASLP*, 2014.
- S. Koyama, *et al.* "Real-time sound field transmission system by using wave field reconstruction filter and its evaluation," *IEICE Trans. Fundam.*, 2014.
- S. Koyama, *et al.* "Analytical approach to wave field reconstruction filtering in spatio-temporal frequency domain," *IEEE Trans. ASLP*, 2013.
- S. Koyama, *et al.* "Reproducing virtual sound sources in front of a loudspeaker array using inverse wave propagator," *IEEE Trans. ASLP*, 2012.