Sound Field Analysis and Synthesis: Theoretical Advances and Applications to Spatial Audio Reproduction

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Sound field analysis/synthesis and its applications



Room acoustic analysis

Signal enhancement

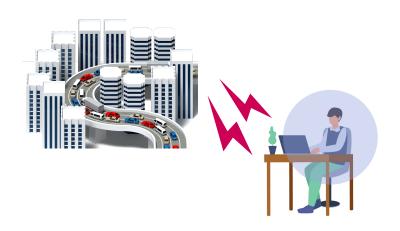
Basic Technologies of Sound Field Analysis and Synthesis



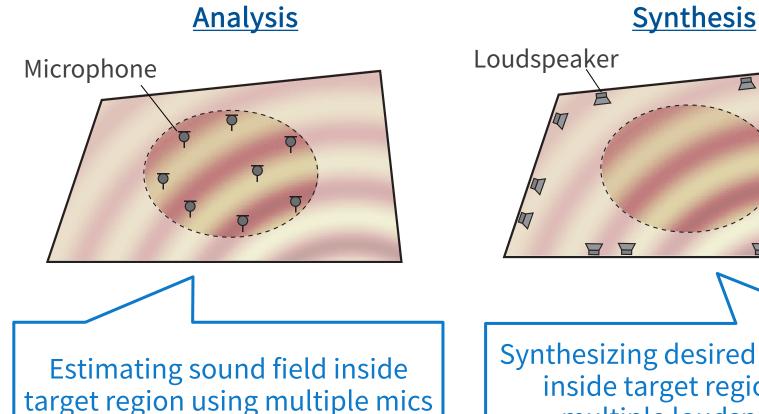
Local-field recording and reproduction

Visualization/auralization

Active noise control



What is sound field analysis/synthesis?



Synthesizing desired sound field inside target region using multiple loudspeakers

Sound field analysis techniques incorporating properties of acoustic field into machine learning techniques

Governing equations in acoustic field

Sound propagation is governed by wave equation in time domian and Helmholtz equation in frequency domain

- \succ Sound pressure u at position $oldsymbol{r} \in \mathbb{R}^3$
 - Wave equation for time t

$$\nabla^2 u(\boldsymbol{r}, t) - \frac{1}{c^2} \frac{\partial^2 u(\boldsymbol{r}, t)}{\partial t^2} = 0$$



– Helmholtz equation for wave number $k=\omega/c$

$$(\nabla^2 + k^2)u(\boldsymbol{r}, k) = 0$$

Hereafter, all the formulations are in frequency domain

Expansion representations of sound field

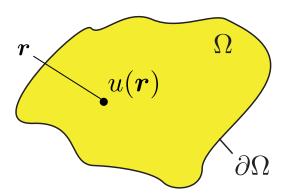
- > Sound field inside source-free interior regin u(r) $(r \in \Omega)$ can be represented by superposition of wave functions
 - Plane wave expansion (a.k.a. Herglotz wave function):

$$u({m r}) = \int_{\mathbb{S}^2} \tilde{u}({m \eta}) \mathrm{e}^{\mathrm{j}k{m \eta}\cdot{m r}} \mathrm{d}{m \eta}^{\mathrm{Plane}}$$
 wave function

Unit vector of propagation direction

Spherical wave function expansion:

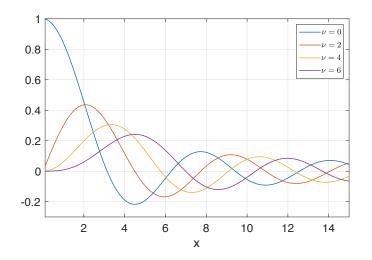
$$u(\boldsymbol{r}) = \sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} \mathring{u}_{\nu,\mu} \sqrt{4\pi} j_{\nu}(kr) Y^{\mu}_{\nu}(\theta,\phi)$$
 Spherical wave function



Expansion representations of sound field

Spherical Bessel function

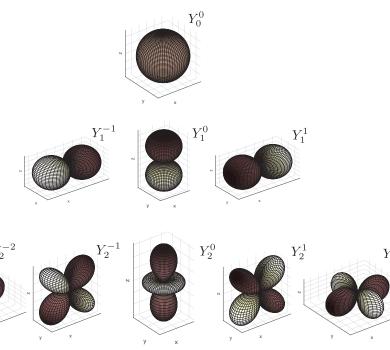
$$j_{
u}(z) = \sqrt{rac{\pi}{2z}} J_{
u+1/2}(z)$$
Bessel function



Spherical harmonic function

$$Y_{\nu}^{\mu}(\theta,\phi) = \sqrt{\frac{(2\nu+1)}{4\pi} \frac{(\nu-\mu)!}{(\nu+\mu)!}} P_{\nu}^{\mu}(\cos\theta) e^{j\mu\phi}$$

Associated Legendre function



Translation in spherical wave function expansion

- \succ Transforming expansion coefficients around $m{r}_0$ into those around $m{r}_0'$
 - Vector of expansion coefficients $\mathring{u}_{\nu,\mu}$ around ${m r}_0$ is defined as $\mathring{u}({m r}_0)\in\mathbb{C}^\infty$
 - $\mathring{m{u}}(m{r}_0')$ and $\mathring{m{u}}(m{r}_0)$ are related as

$$\mathring{\boldsymbol{u}}(\boldsymbol{r}_0') = \boldsymbol{T}(\boldsymbol{r}_0' - \boldsymbol{r}_0)\mathring{\boldsymbol{u}}(\boldsymbol{r}_0)$$

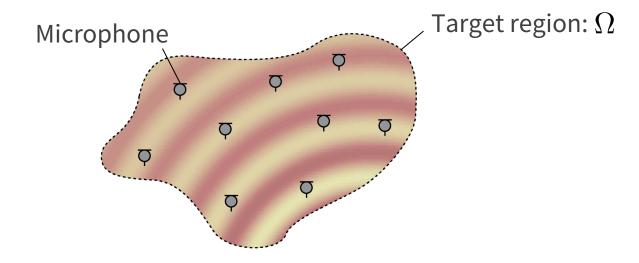
where $T(r_0'-r_0)\in\mathbb{C}^{\infty imes\infty}$ is translation operator defined by

Element of order
$$\nu$$
 and degree μ
$$[\boldsymbol{T}(\boldsymbol{r})\mathring{\boldsymbol{u}}]_{\nu,\mu} = \sum_{\nu'=0}^{\infty} \sum_{\mu'=-\nu'}^{\nu'} \left[4\pi (-1)^{\mu'} \mathrm{j}^{\nu-\nu'} \right.$$

$$\times \sum_{l=0}^{\nu+\nu'} \mathrm{j}^l j_n(kr) Y_l^{\mu-\mu'}(\theta,\phi)^* \mathcal{G}(\nu',\mu';\nu,-\mu,l) \right] \mathring{\boldsymbol{u}}_{\nu',\mu'}$$
 Gaunt coefficient

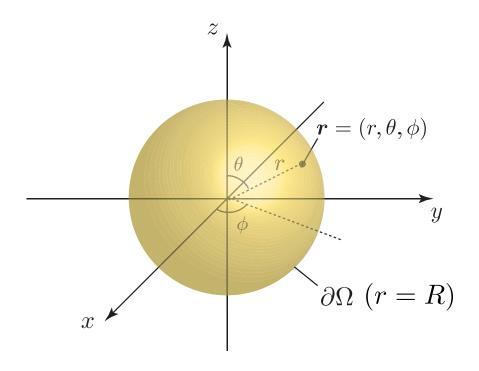
Sound field analysis

Estimate u(r) in target region Ω with observations $\{s_m\}_{m=1}^M$ at discrete set of M mics $\{r_m\}_{m=1}^M$



- Integral-equation-based method
 - Based on discretization of boundary integral equation
- Numerical-optimization-based method
 - Estimation by minimizing some cost function

- Spherical mic array is typically used in sound field analysis for spatial audio recording
 - Goal is to estimate expansion coefficients of spherical wave functions around array center $\{\mathring{u}_{\nu,\mu}\}$ from observations $\{s_m\}_{m=1}^M$ on $\partial\Omega$



- ➤ Integral-equation-based method [Poletti 2005]
 - Spherical harmonic coefficients on $\partial\Omega$ is obtained by

$$U_{\nu,\mu}(R) = \int_0^{2\pi} \int_0^{\pi} u(R,\theta,\phi) Y_{\nu}^{\mu}(\theta,\phi)^* \sin\theta d\theta d\phi$$

lacksquare Discretization by M microphone positions on $\partial\Omega$

$$U_{\nu,\mu}(R) = \sum_m \gamma_m u(R,\theta_m,\phi_m) Y^\mu_\nu(\theta_m,\phi_m)^*$$
 Weight Observation s_m

– Expansion coefficients $\{\mathring{u}_{\nu,\mu}\}$ are estimated by

$$\hat{\dot{u}}_{\nu,\mu} = \frac{1}{\sqrt{4\pi} j_{\nu}(kR)} U_{\nu,\mu}(R)$$

Incomputable when $j_{\nu}(kR) = 0$! (forbidden frequency problem)

- Several techniques for avoiding forbidden frequency problem
 - 1. Mics mounted on rigid spherical baffle
 - 2. Array of directional mics (e.g., unidirectional mics)
 - 3. Two (or more) layers of spherical mic array

1.



mh acoustics em32 Eigenmike®

2.



Core Sound OctoMic™

3.



[Jin+IEEE/ACM TASLP 2014]

 \succ Sound field scattered by rigid spherical baffle of radius R

$$u(\mathbf{r}) = \sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} \mathring{u}_{\nu,\mu} \sqrt{4\pi} \left[j_{\nu}(kr) - \frac{j'_{\nu}(kR)}{h'_{\nu}(kR)} h_{\nu}(kr) \right] Y_{\nu}^{\mu}(\theta,\phi)$$

> Expansion coefficients are estimated by

$$\hat{u}_{\nu,\mu} = \frac{1}{\sqrt{4\pi} \left[j_{\nu}(kR) - \frac{j'_{\nu}(kR)}{h'_{\nu}(kR)} h_{\nu}(kR) \right]} U_{\nu,\mu}(R)$$

$$= -\frac{jk^2 R^2}{\sqrt{4\pi}} h'_{\nu}(kR) U_{\nu,\mu}(R)$$

Much more robust than open spherical mic array

- ➤ Numerical-optimization-based method [Wang+ 1998, Laborie+ 2003]
 - Constructing linear equation of measurements and expansion coefficients by truncating expansion order up to N

$$\begin{bmatrix} s_1 \\ \vdots \\ s_M \end{bmatrix} \approx \begin{bmatrix} \varphi_0^0(\boldsymbol{r}_0) & \cdots & \varphi_N^N(\boldsymbol{r}_0) \\ \vdots & \ddots & \vdots \\ \varphi_0^0(\boldsymbol{r}_M) & \cdots & \varphi_N^N(\boldsymbol{r}_M) \end{bmatrix} \begin{bmatrix} \mathring{u}_{0,0} \\ \vdots \\ \mathring{u}_{N,N} \end{bmatrix}$$

$$\boldsymbol{s} \in \mathbb{C}^M \qquad \boldsymbol{\Phi} \in \mathbb{C}^{M \times (N+1)^2} \qquad \mathring{\boldsymbol{u}} \in \mathbb{C}^{(N+1)^2}$$
where $\varphi_{\nu}^{\mu}(\boldsymbol{r}) = \sqrt{4\pi} j_{\nu}(kr) Y_{\nu}^{\mu}(\theta, \phi)$

Expansion coefficients are estimated as least-squares solution

$$\hat{\ddot{m{u}}} = (m{\Phi}^\mathsf{H}m{\Phi})^{-1}m{\Phi}^\mathsf{H}m{s}$$

Forbidden frequency problem must be avoided in similar way to integral-equation-based method

Summary of conventional analysis methods

Integral-equation-based method

- Stable computation and useful for analyzing properties
- Several well-established methods for avoiding forbidden frequency problem
- Mics must be on spherical surface, and expansion coefficients at array center are obtained

Numerical-optimization-based method

- Arbitrary array geometries are applicable, but expansion center must be set
- Forbidden frequency problem must be avoided when pressure mics are only on boundary surface
- Truncation of expansion order is necessary
- → More accurate and flexible method for sound field analysis?

[Ueno+ IEEE SPL 2018]

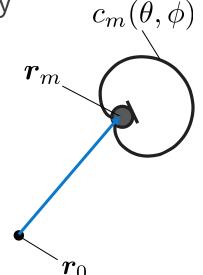
- > Consider to use arbitrarily placed directional mics
 - Expansion coefficients of directivity pattern of m th mic $c_m(\theta,\phi)$ is denoted as $\{c_{m,\nu,\mu}\}$

$$c_{m,\nu,\mu} = \frac{(-j)^{\nu}}{\sqrt{4\pi}} \int_0^{\pi} \int_0^{2\pi} c_m(\theta,\phi)^* Y_{\nu}^{\mu}(\theta,\phi)^* \sin\theta d\theta d\phi$$

- Vectors of $\{\mathring{u}_{\nu,\mu}(\boldsymbol{r}_m)\}$ and $\{c_{m,\nu,\mu}\}$ are denoted as $\mathring{\boldsymbol{u}}(\boldsymbol{r}_m)\in\mathbb{C}^{\infty}$ and $\boldsymbol{c}_m\in\mathbb{C}^{\infty}$, respectively
- Observation of m th mic at \boldsymbol{r}_m is represented by

$$s_m = \sum_{
u=0}^{\infty} \sum_{\mu=-
u}^{
u} c_{m,
u,\mu}^* \mathring{u}_{
u,\mu}(m{r}_m) \ = m{c}_m^\mathsf{H} m{r}(m{r}_m - m{r}_0) \mathring{u}(m{r}_0)$$

Translation operator



[Ueno+ IEEE SPL 2018]

- Modeling of mic observations
 - Observation vector $s \in \mathbb{C}^M$ is written as

$$oldsymbol{s} = oldsymbol{\Xi}(oldsymbol{r}_0)^{\mathsf{H}} \mathring{oldsymbol{u}}(oldsymbol{r}_0)$$

where

$$egin{aligned} egin{aligned} egin{aligned\\ egin{aligned} egi$$

– Sensor noise is assumed to be complex Gaussian $\boldsymbol{\epsilon} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma^2 \boldsymbol{I}_M)$

$$oldsymbol{s} = oldsymbol{\Xi}(oldsymbol{r}_0)^{\mathsf{H}} \mathring{oldsymbol{u}}(oldsymbol{r}_0) + oldsymbol{\epsilon}$$

ightharpoonup Directional mic observations and expansion coefficients around $oldsymbol{r}_0$ are related

[Ueno+IEEE SPL 2018]

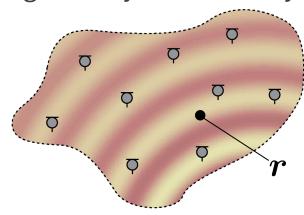
ightharpoonup Expansion coefficients around any position $oldsymbol{r}$ can be estimated as

$$egin{aligned} \hat{m{u}}(m{r}) &= m{T}(m{r}-m{r}_0)m{\Xi}(m{r}_0) \left(m{\Xi}(m{r}_0)^\mathsf{H}m{\Xi}(m{r}_0) + \sigma^{-2}m{I}_M
ight)^{-1}m{s} \ &= m{\Xi}(m{r}) \left(m{\Psi} + \sigma^{-2}m{I}_M
ight)^{-1}m{s} \end{aligned}$$

where (m,m') th element of $\Psi = \Xi(r_0)^H\Xi(r_0) \in \mathbb{C}^{M\times M}$ is obtained by

$$egin{aligned} oxed{\left(\Psi
ight)_{m,m'}} &= oldsymbol{c}_m^\mathsf{H}oldsymbol{T}(oldsymbol{r}_m - oldsymbol{r}_0)oldsymbol{T}(oldsymbol{r}_0 - oldsymbol{r}_{m'})oldsymbol{c}_{m'} \ &= oldsymbol{c}_m^\mathsf{H}oldsymbol{T}(oldsymbol{r}_m - oldsymbol{r}_{m'})oldsymbol{c}_{m'} \ &= oldsymbol{T}(oldsymbol{r} + oldsymbol{r}')oldsymbol{r}(oldsymbol{r}) oldsymbol{r} \ &= oldsymbol{r}(oldsymbol{r})oldsymbol{T}(oldsymbol{r}) oldsymbol{r}(oldsymbol{r}') oldsymbol{r} \ &= oldsymbol{r}(oldsymbol{r})oldsymbol{r}(oldsymbol{r}) oldsymbol{r}(oldsymbol{r}) oldsymbol{r}(oldsymbol{r})$$

Note that $\{c_{m,\nu,\mu}\}$ is generally modelled by low-order coefficients



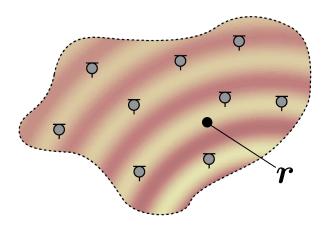
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[Ueno+ IEEE SPL 2018]

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- > Benefits of infinite-dimensional harmonic analysis
 - Arbitrarily placed mics can be used
 - Truncation of expansion order is unnecessary
 - Independent of expansion center

$$\hat{m{u}}(m{r}) = m{\Xi}(m{r}) \left(m{\Psi} + \sigma^{-2}m{I}_M
ight)^{-1} m{s}$$
 $(m{\Psi})_{m,m'} = m{c}_m^{\sf H}m{T}(m{r}_m - m{r}_{m'})m{c}_{m'}$



Estimation of pressure distribution $u(\mathbf{r})$ with pressure (or omnidirectional) mics, i.e., $c_{m,\nu,\mu} = \delta_{\nu,0}\delta_{\mu,0}$

$$\hat{u}(\boldsymbol{r}) = \hat{u}_{0,0}(\boldsymbol{r}) \\
= \left[j_0(k \| \boldsymbol{r} - \boldsymbol{r}_1 \|) \quad \cdots \quad j_0(k \| \boldsymbol{r} - \boldsymbol{r}_M \|) \right] \left(\boldsymbol{\Psi} + \sigma^{-2} \boldsymbol{I}_M \right)^{-1} \boldsymbol{s} \\
= \sum_{m=1}^{M} \left[\left(\boldsymbol{\Psi} + \sigma^{-2} \boldsymbol{I}_M \right)^{-1} \boldsymbol{s} \right]_m j_0(k \| \boldsymbol{r} - \boldsymbol{r}_m \|) \\
\boldsymbol{\Psi} = \begin{bmatrix} j_0(k \| \boldsymbol{r}_1 - \boldsymbol{r}_1 \|) & \cdots & j_0(k \| \boldsymbol{r}_1 - \boldsymbol{r}_M \|) \\ \vdots & \ddots & \vdots \\ j_0(k \| \boldsymbol{r}_M - \boldsymbol{r}_1 \|) & \cdots & j_0(k \| \boldsymbol{r}_M - \boldsymbol{r}_M \|) \end{bmatrix}$$

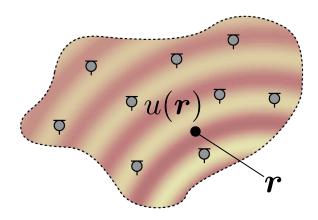
Corresponding to kernel ridge regression with kernel function of 0th-order spherical Bessel function

Consider the following interpolation problem:

$$\begin{aligned} & \underset{u \in \mathcal{H}}{\text{minimize}} \, J := \sum_{m=1}^M |u(\boldsymbol{r}_m) - s_m|^2 + \lambda \|u\|_{\mathcal{H}}^2 \\ & \text{Regularization parameter} \end{aligned}$$

- When solution space \mathcal{H} is reproducing kernel Hilbert space (RKHS), this problem corresponds to kernel ridge regression
- RKHS is Hilbert space $(\mathcal{H}, \langle \cdot, \cdot \rangle_{\mathcal{H}})$ that has reproducing kernel function $\kappa : \mathcal{H} \times \mathcal{H} \to \mathbb{C}$

$$u(\mathbf{r}) = \langle \kappa(\cdot, \mathbf{r}), u \rangle_{\mathcal{H}}$$



- Based on representer theorem, the problem has closed-form solution
 - The solution is represented with $\alpha_m \in \mathbb{C} \ (m=1,\ldots,M)$ as

$$u(\mathbf{r}) = \sum_{m=1}^{M} \alpha_m \kappa(\mathbf{r}, \mathbf{r}_m)$$

– Vector of $\alpha = [\alpha_1, \dots, \alpha_M]^\mathsf{T} \in \mathbb{C}^M$ is obtained by

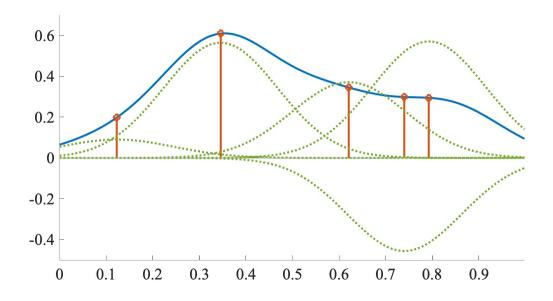
$$\boldsymbol{lpha} = (\boldsymbol{K} + \lambda \boldsymbol{I}_M)^{-1} \boldsymbol{s}$$

where $\mathbf{K} \in \mathbb{C}^{M \times M}$ is Gram matrix defined as

$$m{K} := egin{bmatrix} \kappa(m{r}_1,m{r}_1) & \cdots & \kappa(m{r}_1,m{r}_M) \ dots & \ddots & dots \ \kappa(m{r}_M,m{r}_1) & \cdots & \kappa(m{r}_M,m{r}_M) \end{bmatrix}$$

- ➤ Interpolation is achieved by linear combination of kernel functions at data points
 - Typically-used kernel function in machine learning is Gaussian kernel

$$\kappa(\boldsymbol{r}_1, \boldsymbol{r}_2) = \exp\left(-rac{\|\boldsymbol{r}_1 - \boldsymbol{r}_2\|^2}{\sigma^2}
ight)$$



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[Ueno+ IWAENC 2018]

- RKHS for interpolating sound field
 - $-(\mathcal{H},\langle\cdot,\cdot\rangle_{\mathcal{H}})$ is defined as

$$\mathcal{H} := \left\{ \int_{\mathbb{S}^2} \tilde{u}(\boldsymbol{\eta}) e^{jk\boldsymbol{\eta}\cdot\boldsymbol{r}} d\boldsymbol{\eta} \mid \tilde{u} \in L_2(\mathbb{S}^2) \right\}$$
$$\langle u_1, u_2 \rangle_{\mathcal{H}} := \int_{\mathbb{S}^2} \tilde{u}(\boldsymbol{\eta})^* \tilde{u}(\boldsymbol{\eta}) d\boldsymbol{\eta}$$

Any solution of homogeneous Helmholtz equation can be approximated arbitrarily by functions in ${\cal H}$

Reproducing kernel function for sound field interpolation

$$\kappa(\boldsymbol{r}_1, \boldsymbol{r}_2) = \int_{\mathbb{S}^2} e^{jk\boldsymbol{\eta}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)} d\boldsymbol{\eta}$$
$$= j_0(k||\boldsymbol{r}_1 - \boldsymbol{r}_2||)$$

[Ueno+ IWAENC 2018]

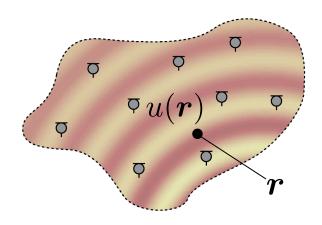
Pressure distribution is estimated by

$$\hat{u}(\boldsymbol{r}) = \left[(\boldsymbol{K} + \lambda \boldsymbol{I}_M)^{-1} \boldsymbol{s} \right]^{\mathsf{T}} \boldsymbol{\kappa}(\boldsymbol{r})$$

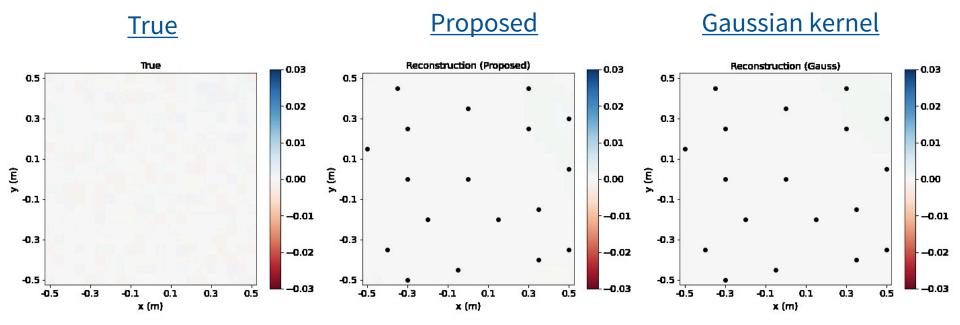
where

$$oldsymbol{K} := egin{bmatrix} j_0(k \|oldsymbol{r}_1 - oldsymbol{r}_1\|) & \cdots & j_0(k \|oldsymbol{r}_1 - oldsymbol{r}_M\|) \ dots & \ddots & dots \ j_0(k \|oldsymbol{r}_M - oldsymbol{r}_1\|) & \cdots & j_0(k \|oldsymbol{r}_M - oldsymbol{r}_M\|) \end{bmatrix}$$

$$\boldsymbol{\kappa}(\boldsymbol{r}) := \begin{bmatrix} j_0(k\|\boldsymbol{r}-\boldsymbol{r}_1\|) & \cdots & j_0(k\|\boldsymbol{r}-\boldsymbol{r}_M\|) \end{bmatrix}^\mathsf{T}$$



- > Experimental results using real data
 - Reconstructing pulse signal from single loudspeaker w/ 18 mic



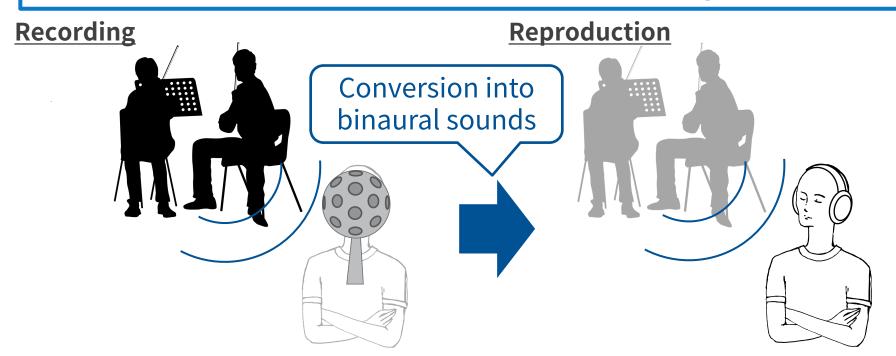
(Black dots indicate mic positions)



Impulse response measurement system

[Koyama+ IEEE WASPAA 2021]

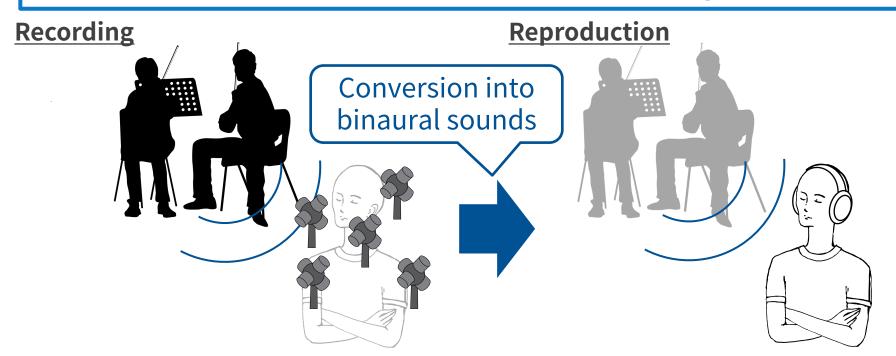
Binaural reproduction from mic array recordings for VR audio



- ➤ Binaural reproduction in real world is difficult, comparing with binaural synthesis in VR space
- Conventional spherical array processing requires largescale system to achieve broad listening area

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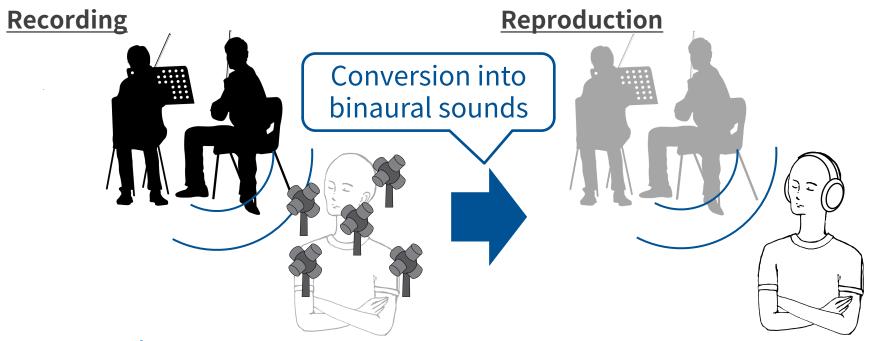
Binaural reproduction from mic array recordings for VR audio



- Binaural reproduction from recordings of multiple small arrays instead of single spherical array
- Broad listening area by using flexible and scalable recording system

[lijima+ JASA 2021]

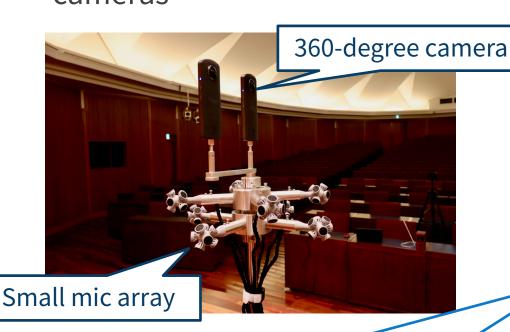
Binaural reproduction from mic array recordings for VR audio

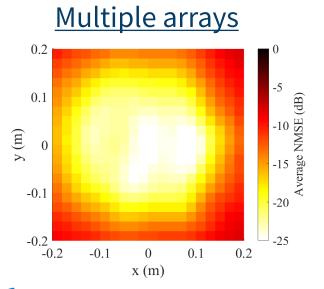


- Recording
 - Sound field estimation by infinite-dimensional harmonic analysis
- Reproduction
 - Binaural rendering from estimated harmonic coefficients with compensation for loudspeaker distance in HRTF measurements

[lijima+ JASA 2021]

Recording system using multiple small arrays and 360-degree cameras

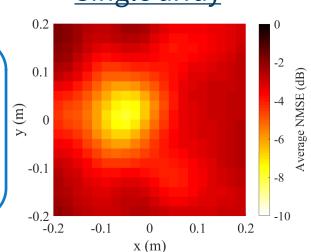




Binaural reproduction error distribution

- High accuracy in broad region
- Rotational and translational head movement is possible (3DoF → 6DoF)







- > Demo available on Youtube
 - https://youtu.be/tsGIITmQiug
 - Recorded classical music (String Quartet)
 - 360-degree control of viewing and listening directions
 - 2 viewing and listening positions are selectable



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Conclusion

- > Recent advances in sound field analysis and synthesis
 - Two categories of sound field analysis methods:
 - Integral-equation-based method
 - Numerical-optimization-based method
 - Introduced infinite-dimensional harmonic analysis
 - Arbitrarily placed mics can be used
 - Truncation of expansion order is unnecessary
 - Independent of expansion center
 - Kernel interpolation of sound field as a special case of infinite-dimensional harmonic analysis
 - Application to binaural reproduction from mic array recordings

Conclusion

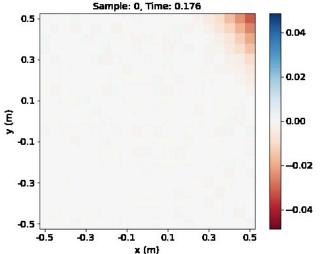
> Related works:

- Incorporating prior knowledge on approximate source direction by using directional weighting in kernel function to enhance accuracy [Ueno+ IEEE TSP 2021]
- Adaptation of directional weighting in kernel function from observation data [Horiuchi+ IEEE WASPAA 2021]
- Application to sound field reproduction by using multiple loudspeakers [Koyama+ I3DA 2021]
- Application to spatial active noise control [Koyama+ IEEE/ACM TASLP 2021]
- Example codes of sound field analysis and synthesis techniques are available with room impulse response dataset [Koyama+ IEEE WASPAA 2021]

Dataset of room impulse responses (RIRs)

- Released RIR dataset on meshed grid points with example codes
 - https://sh01k.github.io/MeshRIR/







RIR measurement system

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